**Gambling using the Martingale strategy:**

**Numberphile**

*Mathematics applied to real life situations can be intriguing.*

Maths can help alter the perspective of the world we all live in, either by offering a different point of view, allowing you to peer in on the world through the lens of an unbiased bystander, or simply opening up new avenues of thinking, creating new and intriguing thought problems that could range from questioning the laws of physics that govern scientific thinking; to how mathematicians gamble and how they can swing the odds.

**Imagine a fictional game of roulette**, where the only options are red or black. A 50/50 chance that if guessed correctly will double the players money. Now, any standard player could play roulette and win on average 50% of the time. Meaning if they bet the same amount of money every time, they should eventually break even excluding certain situations where they run out of money (which we will touch on later).

But a mathematician isn’t a standard player in any game and neither was Martingale.

His theory says that, by doubling your bet every time you lose you will eventually win. We can show this using a table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Step 1 | Step 2 | Step 3 | Step 4 | Step K |
| £1 | £2 | £4 | £8 | £ |

Here the bets are all increasing powers of 2 ( , etc) so we can derive a formular for the bet at step K (which is just a step placeholder) of

Now, if you lose at step K the total loss is the amount you just bet () plus the sum of all of the previous bets. So 1+2+4+8+…+. This can be written in more mathematical terms using sigma notation as **the sum from J=0 to K-1 of :**

(We use sigma here as it easily represents the sum of previous losses until our required step)

Now this looks confusing but all it is saying is that starting with 0, loss is + + as J increases by one every time. 1 + 2^1 (2) + 2^2(4) and so on.

**This is a geometric series**, which means that we can use the formular for the sum of a geometric series to work out the total loss at step K. The formular for the sum = . Where N= number of terms, A= first term and R = common ratio between terms.

This gives us the equation: which can be simplified to give us the **Total Loss = (**be careful not to confuse this with the loss at step K which is )

Now we have a formular for total loss we can use it to prove we always make a profit: First, as assuming we lost at step K, our next bet will be , which if we win, we earn () profit. This after deductions of our total losses ( ) gives us £1 true profit. Here’s an example: at step 3 with a bet of pounds, our next bet is pounds at (step 4). By winning here we earnt £16 (we then – the previous bets) £16 - £8 -£4 -£2 -£1 = £1 pound profit.

This means that theoretically using the martingale strategy you can guarantee profit every time.

So why aren’t people doing this?

**But this is very theoretical**. Casinos know about the Martingale Strategy and that’s why roulette wheels have one or even 2, 0’s which are neither black or red which just turns the odds even further in their favour. Furthermore, its why casinos have maximum bet limits, meaning that you are unlikely to be able to retain larger profit.

And also, £1 isn’t a lot of profit. But we can do is change the rates. It’s actually the first bet which determines profit if you continue to double your bet every time. So, if you started with an initial bet of 1 million your profit would then be 1 million. But the problem with this is that it will scale up very fast. For instance, a starting bet of a million would only need 12 incorrect bets for richest man Elon Musk (£200 B) to be out of money…

However, where it gets interesting we when we look at how many losses different “balances” allows. As if we brought £100 into the casino (and we couldn’t spend any earnings) it would only take 7 losses in a row to lose it all. And while that may seem like a small amount, it occurs 1/128 times and in a casino with around 500 people it will happen to around 4 of them. But when bringing 10\* more money to the casino or £1000, we only gain 3 more losses until bankruptcy; and this is to still only win £1! And it doesn’t increase much further after that:

|  |  |  |  |
| --- | --- | --- | --- |
| £100 | £1,000 | £10,000 | £1,000,000 |
| 7 losses | 10 | 13 | 20 |

We can work out the probability of us losing all our money depending on our balance when we came into the casino. The probability of losing K times in a row is as the more games we play the less likely our losing streak continues.

We can also work out number of losses we can afford by comparing our bet to our balance. The equation < N shows that you can never bet more than your balance using the equation as It says the bet must always be below your balance. From here let’s set N equal to(N=) which allows us to compare values easier.

Now, by combining our two equations: The probability of losing K times in a row being and N= we get an equation that states the probability of losing K times so that you can’t bet again is 1/N as N =. If that’s difficult to visualise, imagine walking in with £15, if you lost until you couldn’t bet double again would have had to lose 4 times. This is the same as 1/16 or using N as the denominator is 1/15.

From this, we can say that the probability of you winning 1 pound by using the strategy is 1 – 1/N. As if the possibility of losing every time is 1/N the probability of winning once is 1- this.

From this we can work out the probability of doubling your money: As if you wanted to win £100 using this strategy, the equation would be (1-1/N)^100. This is as this calculation has to work 100 times in a row without failure.

So doubling your money (for any N) is (1-1/N)^N remember this is starting from £1. We can also see intriguing results from the table:

|  |  |  |  |
| --- | --- | --- | --- |
| N =100 | 1000 | 1000000 | ∞ |
| 36.6% | 36.8% | 36.8% | 36.8% |

This is the best bit. As your “N” becomes larger and larger you eventually reach a **constant of 36.8%** This is also known as **1/E which is Euler’s number,** the natural rate of growth. It shouldn’t be a surprise that Eulers number shows up here, as the constant also appears commonly in compound interest, probability and other forms of growth over time, including money.

And so, it leads to the Question, if the chance of doubling your money is only ever going to be around 37% by applying this Martingale strategy:

**Why don’t you just put everything on the first spin for a 50/50 chance?**

And so, we realise that while this gambling strategy and ones alike, will eventually break down due lack of infinite money and so the more time goes on, the further the value falls to 0.

And maybe this should have been obvious from the get-go, as Martingale was the owner of a British gambling house in which he promoted the use of his system so when you consider that he would benefit from his customers going bankrupt, this should give you an idea of who the house ultimately favours.

But there’s other alternatives to the martingale like the anti-martingale, where you only double after wins, meaning that if you go on a decent winning streak you can earn a fair amount of money. But to still requires you to win.